Lecture 4: An introduction to input-output analysis and its use in consumption-based accounting

Dr Anne Owen
a.owen@leeds.ac.uk
@dr_anneowen
Homework: Introduction to Matrix Algebra using Matlab

• Any problems?

• The essentials:
  • Adding and subtracting matrices
  • Matrix multiplication has two forms – product and dot product
  • What happens when you multiply a matrix by its inverse?
Lecture (45 mins)

- What does a consumption-based approach to emissions accounting tell us?

- How can we account for emissions from consumption?

- Input-output analysis
Carbon emissions are increasing, but what are the drivers of this increase?

- Increase in industrial output?
- Population?
- Increase in air travel?
- Increase in consumption?
- Production becoming more inefficient?
Emissions from OECD countries are about the same as in 1990. Emissions from non-OECD countries have increased rapidly in the last decade.
Countries have a broad range of per capita emissions reflecting their national circumstances.
Cumulative emissions from fossil-fuel and industry were distributed (1870–2017): USA 26%, EU28 22%, China 13%, Russia 7%, Japan 4% and India 3%

Historically, the EU and US still contribute the greatest share of emitted CO₂
Where are the world’s toys made?
Who buys the toys?
The UK

![Graph showing CO2 emissions in the UK from 1997 to 2015. The graph compares UK consumption-based accounts and UK production. The emissions have generally decreased over the years.](image-url)
Allocating fossil and industry emissions to the consumption of products provides an alternative perspective.

USA and EU28 are net importers of embodied emissions, China and India are net exporters.

Consumption-based emissions are calculated by adjusting the standard production-based emissions to account for international trade.
Flows from location of generation of emissions to location of consumption of goods and services
2 minute discussion

How can we assign emissions responsibility to consumers?

What do we need to measure?
What volume of GHGs are emitted by which industries?
Countries report their emissions by industry to the UNFCCC

How are products made?
Need to understand production recipes and global trade

How are products bought?
Spend by households and governments by product is used to calculate GDP
What are the GHG emissions associated with spending £38 on fruit and veg?

Stage 0

Fruit and veg industry uses 5 tonnes GHG to produce £92 of fruit and veg output.

\[ e_{\text{fruit \\& veg}} = 0.05 \text{ tGHG/£.} \]

\( e \) is a measure of emissions intensity

Spend on fruit and veg \((y_{\text{fruit \\& veg}})\) is £38

Impact is \(38 \times 0.05 = 2.06\) tGHG.

\( y \) is a measure of final demand

Note – we find the impact of £1 and multiply by the spend

BUT! What about the rest of the supply chain?? There are other stages involved in the production of a fruit and veg product! The impact is more than 2.06 tGHG
This is an input output table!

<table>
<thead>
<tr>
<th></th>
<th>Fruit and Veg</th>
<th>Fertiliser</th>
<th>Tractor</th>
<th>Wheels</th>
<th>Rubber</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fruit and Veg</strong></td>
<td>50</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Fertiliser</strong></td>
<td>10</td>
<td>20</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Tractor</strong></td>
<td>10</td>
<td>1</td>
<td>25</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Wheels</strong></td>
<td>1</td>
<td>1</td>
<td>20</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td><strong>Rubber</strong></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td><strong>Taxes &amp; wages</strong></td>
<td>20</td>
<td>30</td>
<td>15</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>92</td>
<td>44</td>
<td>63</td>
<td>48</td>
<td>54</td>
</tr>
<tr>
<td><strong>GHGs</strong></td>
<td>5</td>
<td>10</td>
<td>12</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

**Where are items sold to?**

**Who else buys products?**

<table>
<thead>
<tr>
<th></th>
<th>Hhold &amp; Govt spend</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fruit and Veg</strong></td>
<td>38</td>
<td>92</td>
</tr>
<tr>
<td><strong>Fertiliser</strong></td>
<td>11</td>
<td>54</td>
</tr>
<tr>
<td><strong>Tractor</strong></td>
<td>25</td>
<td>63</td>
</tr>
<tr>
<td><strong>Wheels</strong></td>
<td>10</td>
<td>48</td>
</tr>
<tr>
<td><strong>Rubber</strong></td>
<td>1</td>
<td>54</td>
</tr>
</tbody>
</table>

Here is the £38 spent on fruit and veg

Here is the 5 tonnes of GHGs emitted by the fruit and veg industry
First let's use the production recipe to find out what else is needed to make £1 of fruit and veg. The table of proportions is:

<table>
<thead>
<tr>
<th></th>
<th>Fruit and Veg</th>
<th>Fertiliser</th>
<th>Tractor</th>
<th>Wheels</th>
<th>Rubber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fruit and Veg</td>
<td>0.54</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Fertiliser</td>
<td>0.11</td>
<td>0.45</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Tractor</td>
<td>0.11</td>
<td>0.02</td>
<td>0.40</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Wheels</td>
<td>0.01</td>
<td>0.02</td>
<td>0.32</td>
<td>0.31</td>
<td>0.02</td>
</tr>
<tr>
<td>Rubber</td>
<td>0.01</td>
<td>0.02</td>
<td>0.42</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>Taxes &amp; wages</td>
<td>0.22</td>
<td>0.45</td>
<td>0.24</td>
<td>0.21</td>
<td>0.37</td>
</tr>
<tr>
<td>Total</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>GHGs</td>
<td>0.05</td>
<td>0.23</td>
<td>0.19</td>
<td>0.15</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Stage 1 of the supply chain to make £1 of fruit and veg involves:

- 0.54 x 0.05 (fruit & veg)
- 0.11 x 0.23 (fertiliser)
- 0.11 x 0.19 (tractor)
- 0.01 x 0.15 (wheels)
- 0.01 x 0.15 (rubber)

= 0.078 tGHGs

So £38 is 2.97 tGHGs
Now we have £38 of spend on fruit and veg producing:

2.06 tGHG in Stage 0 (on site)

2.97 tGHG in Stage 1 (emissions from intermediate industries serving the fruit and veg industry)
BUT! The tractor bought by the fruit & veg industry also has a supply chain!

And the fertiliser has to be made...

And the tractor has to have its wheels made...

Stage 2 of the supply chain to make £0.11 of tractor involves:

\[ 0.11 \times 0.02 \times 0.05 + 0.11 \times 0.02 \times 0.23 + 0.11 \times 0.40 \times 0.19 + 0.11 \times 0.32 \times 0.15 + 0.11 \times 0.02 \times 0.15 = 0.014 \text{ tGHGs} \]

So £38 is 0.54 tGHGs
Now we have £38 of spend on fruit and veg producing:

2.06 tGHG in Stage 0 (on site)

2.97 tGHG in Stage 1 (emissions from intermediate industries serving the fruit and veg industry)

0.54 tGHG in Stage 2 (emissions from making the tractor used by the fruit and veg industry)

The full supply chain emissions (tracing all paths) for £38 of fruit and veg is actually 15.48 tGHG!
What does this look like in algebra?

\[ Q_1 = e_1 \cdot y_1 + e_1 \cdot A_{11} \cdot y_1 + e_2 \cdot A_{21} \cdot y_1 + e_3 \cdot A_{31} \cdot y_1 + e_4 \cdot A_{41} \cdot y_1 + e_5 \cdot A_{41} \cdot y_1 + e_1 \cdot A_{11} \cdot A_{11} \cdot y_1 + e_2 \cdot A_{21} \cdot A_{11} \cdot y_1 + e_3 \cdot A_{31} \cdot A_{11} \cdot y_1 + \text{etc} \]

More simply

\[
Q = eIy + eAy + eA^2y + eA^3y + ... + eA^ny \\
Q = e(I + A + A^2 + A^3 + ... + A^n)y \\
Q = e(I - A)^{-1}y \\
Q = eLy
\]

This is the environmentally-extended Leontief inverse equation!
What volume of GHGs are emitted by which industries?

How are products made? Which industries are involved?

How are products bought? Who buys them? From where?

Row vector of emissions per £ of industrial output

Leontief Inverse – a matrix which reallocates money on industrial production to money on consumption of products

Final demand vector showing spend by final consumers

Consumption emissions

\[ e \times L \times y = Q \]
But our fruit and veg example did not have any importing countries in it...

<table>
<thead>
<tr>
<th>Country</th>
<th>A’s Domestic transactions</th>
<th>Country</th>
<th>A’s industry used to make Country B’s products</th>
<th>Country</th>
<th>A’s industry used to make Country C’s products</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B’s Domestic transactions</td>
<td>B</td>
<td>B’s Domestic transactions</td>
<td>B</td>
<td>B’s Domestic transactions</td>
</tr>
<tr>
<td>C</td>
<td>C’s Domestic transactions</td>
<td>C</td>
<td>C’s Domestic transactions</td>
<td>C</td>
<td>C’s Domestic transactions</td>
</tr>
</tbody>
</table>

A multi-regional input output database

<table>
<thead>
<tr>
<th>VA from A</th>
<th>VA from B</th>
<th>VA from C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>C’s FD from A</td>
</tr>
<tr>
<td>B’s FD from A</td>
</tr>
<tr>
<td>A’s FD from A</td>
</tr>
</tbody>
</table>

Total output
2 minute discussion

Can you think of any issues there might be in constructing a table using data from every country in the world?
School of Earth & Environment
5582: Tools and Techniques in Ecological Economics

• Issues
• Sectoral classifications & meaning
• Currencies
• Bilateral trade mismatches
• Missing data
WINTER IS COMING
• Consider a world with two countries: Westeros and Essos

• Each country has 3 industries: food, machines, energy

• Let’s construct a multi-regional input-output database!
<table>
<thead>
<tr>
<th></th>
<th>Westeros Food</th>
<th>Westeros Machines</th>
<th>Westeros Energy</th>
<th>Essos Food</th>
<th>Essos Machines</th>
<th>Essos Energy</th>
<th>Westeros Demand</th>
<th>Essos Demand</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Westeros Food</td>
<td>660</td>
<td>10</td>
<td>2</td>
<td>30</td>
<td>1</td>
<td>1</td>
<td>690</td>
<td>60</td>
<td>1454</td>
</tr>
<tr>
<td>Westeros Machines</td>
<td>300</td>
<td>500</td>
<td>100</td>
<td>20</td>
<td>30</td>
<td>20</td>
<td>380</td>
<td>20</td>
<td>1370</td>
</tr>
<tr>
<td>Westeros Energy</td>
<td>200</td>
<td>400</td>
<td>300</td>
<td>10</td>
<td>30</td>
<td>100</td>
<td>330</td>
<td>15</td>
<td>1385</td>
</tr>
<tr>
<td>Essos Food</td>
<td>40</td>
<td>2</td>
<td>1</td>
<td>440</td>
<td>20</td>
<td>2</td>
<td>200</td>
<td>580</td>
<td>1285</td>
</tr>
<tr>
<td>Essos Machines</td>
<td>20</td>
<td>200</td>
<td>200</td>
<td>140</td>
<td>400</td>
<td>90</td>
<td>90</td>
<td>200</td>
<td>1340</td>
</tr>
<tr>
<td>Essos Energy</td>
<td>10</td>
<td>30</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>100</td>
<td>280</td>
<td>1220</td>
</tr>
</tbody>
</table>

Value added       | 224           | 228               | 582             | 445        | 659            | 807          |

Total                  | 1454          | 1370              | 1385            | 1285       | 1340           | 1220         |

GHG                   | 980           | 1000              | 1200            | 1000       | 1100           | 1550         |

What are the production-based emissions for Westeros?

What are the production-based emissions for Essos?

What happens when we find the consumption-based emissions for each nation?
This is $\mathbf{A}$ but we need to calculate $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$

Now we have $\mathbf{y}_w$ and $\mathbf{y}_e$ and we will use each vector to find the CBA of each nation.

We have calculated the emissions intensity $\mathbf{e}$, here. Note how different the intensities are.
\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.45 & 0.01 & 0.00 & 0.02 & 0.00 & 0.00 \\
0.21 & 0.36 & 0.07 & 0.02 & 0.02 & 0.02 \\
0.14 & 0.29 & 0.22 & 0.01 & 0.02 & 0.08 \\
0.03 & 0.00 & 0.00 & 0.34 & 0.01 & 0.00 \\
0.01 & 0.15 & 0.14 & 0.11 & 0.30 & 0.07 \\
0.01 & 0.02 & 0.14 & 0.16 & 0.15 & 0.16 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.55 & -0.01 & 0.00 & -0.02 & 0.00 & 0.00 \\
-0.21 & 0.64 & -0.07 & -0.02 & -0.02 & -0.02 \\
-0.14 & -0.29 & 0.78 & -0.01 & -0.02 & -0.08 \\
-0.03 & 0.00 & 0.00 & 0.66 & -0.01 & 0.00 \\
-0.01 & -0.15 & -0.14 & 0.70 & -0.07 & -0.11 \\
-0.01 & -0.02 & -0.14 & -0.16 & -0.15 & 0.84 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.55 & -0.01 & 0.00 & -0.02 & 0.00 & 0.00 \\
-0.21 & 0.64 & -0.07 & -0.02 & -0.02 & -0.02 \\
-0.14 & -0.29 & 0.78 & -0.01 & -0.02 & -0.08 \\
-0.03 & 0.00 & 0.00 & 0.66 & -0.01 & 0.00 \\
-0.01 & -0.15 & -0.14 & 0.70 & -0.07 & -0.11 \\
-0.01 & -0.02 & -0.14 & -0.16 & -0.15 & 0.84 \\
\end{bmatrix}
\begin{bmatrix}
\end{bmatrix}
\]

\[
\begin{bmatrix}
1.85 & 0.03 & 0.01 & 0.07 & 0.01 & 0.00 \\
0.69 & 1.69 & 0.18 & 0.09 & 0.07 & 0.06 \\
0.61 & 0.67 & 1.39 & 0.11 & 0.10 & 0.16 \\
0.09 & 0.02 & 0.01 & 1.53 & 0.04 & 0.01 \\
0.34 & 0.52 & 0.36 & 0.32 & 1.50 & 0.18 \\
0.22 & 0.26 & 0.31 & 0.36 & 0.29 & 1.26 \\
\end{bmatrix}
\]
So now we have all the component parts we need!

\[ Q_w = eL y_w \]
\[ Q_e = eL y_e \]

But... matrices don’t quite work so easily
Matrix of emissions per £ of industrial output
(put emissions intensity on the diagonal)

Leontief Inverse – a matrix which reallocates money on industrial production to money on consumption of products

Final demand matrix showing spend by final consumers
(put each final demand vector on a diagonal)

\[
\hat{e} \times L \times \hat{y} = Q
\]

Column sum gives product impacts
Row sum gives source impacts
School of Earth & Environment
5582: Tools and Techniques in Ecological Economics

• Exercises 1-3 (20 mins)
  • Create the data needed for the Game of Thrones input-output table
  • Make A the total requirements matrix
  • Make L the Leontief inverse

http://www.see.leeds.ac.uk/teaching-resources/
• Exercise 4 to 5 (40 mins)
  • Generate consumption-based emissions for Westeros and Essos

\[ Q_w = \hat{e}_w\hat{L}_w \]

\[ Q_e = \hat{e}_e\hat{L}_e \]

<table>
<thead>
<tr>
<th>Emissions in Westeros to make Westeros products bought by Westeros</th>
<th>Emissions in Westeros to make Essos products bought by Westeros</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissions in Essos to make Westeros products bought by Westeros</td>
<td>Emissions in Essos to make Essos products bought by Westeros</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Emissions in Westeros to make Essos products bought by Essos</th>
<th>Emissions in Essos to make Essos products bought by Essos</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissions in Essos to make Westeros products bought by Essos</td>
<td>Emissions in Essos to make Westeros products bought by Essos</td>
</tr>
</tbody>
</table>
**Exercise 7 (30 mins)**

- Generate graphs of Westeros and Essos’ consumption-based accounts

\[
Q_w = \hat{\epsilon}_w \hat{L} \hat{y}_w
\]

<table>
<thead>
<tr>
<th>Emissions in Westeros to make Westeros products bought by Westeros</th>
<th>Emissions in Westeros to make Essos products bought by Westeros</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissions in Essos to make Westeros products bought by Westeros</td>
<td>Emissions in Essos to make Essos products bought by Westeros</td>
</tr>
</tbody>
</table>

\[
Q_e = \hat{\epsilon}_e \hat{L} \hat{y}_e
\]

<table>
<thead>
<tr>
<th>Emissions in Westeros to make Westeros products bought by Essos</th>
<th>Emissions in Westeros to make Essos products bought by Essos</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissions in Essos to make Westeros products bought by Westeros</td>
<td>Emissions in Essos to make Essos products bought by Essos</td>
</tr>
</tbody>
</table>
Thank you

Dr Anne Owen
a.owen@leeds.ac.uk
@dr_anneowen